Profesor Blaga Mirela-Gabriela

Limite remarcabile de şiruri

1) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

2) $\lim_{n\to\infty}\left(1+\frac{1}{x_n}\right)^{x_n} = e \qquad , x_n \to \pm\infty$

3) $\lim_{n\to\infty}(1+x_n)^{\frac{1}{x_n}} = e \qquad , x_n \in \mathbb{R}^*, x_n \to 0$

4) $\lim_{n\to\infty}\frac{\ln(1+x_n)}{x_n} = 1 \qquad , x_n \in \mathbb{R}^*, x_n \to 0, 1+x_n > 0$

5) $\lim_{n\to\infty}\frac{a^{x_n}-1}{x_n} = ln\, a \qquad , x_n \in \mathbb{R}^*, x_n \to 0, a > 0$

6) $\lim_{n\to\infty}\frac{(1+x_n)^r-1}{x_n} = r \quad , x_n \in \mathbb{R}^*, x_n \to 0, r \in \mathbb{R}$

7) $\lim_{n\to\infty}\frac{sinx_n}{x_n} = 1 \qquad , x_n \in \mathbb{R}^*, x_n \to 0$

8) $\lim_{n\to\infty}\frac{tgx_n}{x_n} = 1 \qquad , x_n \in \mathbb{R}^*, x_n \to 0$

9) $\lim_{n\to\infty}\frac{arcsinx_n}{x_n} = 1 \qquad , x_n \in \mathbb{R}^*, x_n \to 0$

10) $\lim_{n\to\infty}\frac{arctgx_n}{x_n} = 1 \qquad , x_n \in \mathbb{R}^*, x_n \to 0$

Exemple. Calculaţi limitele:

1) $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

2) $\lim_{n\to\infty}\left(1+\frac{1}{2n+1}\right)^{2n+1} = e$

3) $\lim_{n\to\infty}\left(1+\frac{ln\, n}{n}\right)^{\frac{n}{ln\, n}} = e \quad , \frac{ln\, n}{n}\xrightarrow[n\to\infty]{} 0$

4) $\lim\limits_{n\to\infty} \dfrac{\ln(1+\frac{1}{n})}{\frac{1}{n}} = 1 \quad , \frac{1}{n} \xrightarrow[n\to\infty]{} 0$

5) $\lim\limits_{n\to\infty} \sqrt{n}\left(2^{\frac{1}{\sqrt{n}}} - 1\right) = \lim\limits_{n\to\infty} \dfrac{2^{\frac{1}{\sqrt{n}}} - 1}{\frac{1}{\sqrt{n}}} = ln\, 2 \quad , \frac{1}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$

6) $\lim\limits_{n\to\infty} \dfrac{n}{sin\, n}\left(\sqrt{1+\frac{sin\, n}{n}} - 1\right) = \lim\limits_{n\to\infty} \dfrac{\left(1+\frac{sin\, n}{n}\right)^{\frac{1}{2}} - 1}{\frac{sin\, n}{n}} = \dfrac{1}{2} \quad , \frac{sin\, n}{n} \xrightarrow[n\to\infty]{} 0$

7) $\lim\limits_{n\to\infty} n \cdot sin\frac{1}{n} = \lim\limits_{n\to\infty} \dfrac{sin\frac{1}{n}}{\frac{1}{n}} = 1 \quad , \frac{1}{n} \xrightarrow[n\to\infty]{} 0$

8) $\lim\limits_{n\to\infty} \sqrt{n} \cdot tg\frac{1}{\sqrt{n}} = \lim\limits_{n\to\infty} \dfrac{tg\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = 1 \quad , \frac{1}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$

9) $\lim\limits_{n\to\infty} \dfrac{arcsin\frac{1}{2n+1}}{\frac{1}{2n+1}} = 1 \quad , \frac{1}{2n+1} \xrightarrow[n\to\infty]{} 0$

10) $\lim\limits_{n\to\infty} n^2 \cdot arctg\frac{1}{n^2+1} = \lim\limits_{n\to\infty} \underbrace{n^2 \cdot \frac{1}{n^2+1}}_{1} \cdot \underbrace{\dfrac{arctg\frac{1}{n^2+1}}{\frac{1}{n^2+1}}}_{1} = 1 \cdot 1 = 1 \quad , \frac{1}{n^2+1} \xrightarrow[n\to\infty]{} 0$