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Limite remarcabile de funcţii

1) $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

2) $\lim\limits_{x\to-\infty}\left(1+\frac{1}{x}\right)^x = e$

3) $\lim\limits_{x\to0}(1+x)^{\frac{1}{x}} = e$

4) $\lim\limits_{x\to0}\frac{\ln(1+x)}{x} = 1$

5) $\lim\limits_{x\to0}\frac{a^x-1}{x} = ln\,a \qquad , a>0$

6) $\lim\limits_{x\to0}\frac{e^x-1}{x} = 1$

7) $\lim\limits_{x\to0}\frac{(1+x)^r-1}{x} = r \ , r\in\mathbb{R}$

8) $\lim\limits_{x\to0}\frac{sin\,x}{x} = 1$

9) $\lim\limits_{x\to0}\frac{tg\,x}{x} = 1$

10) $\lim\limits_{x\to0}\frac{arcsin\,x}{x} = 1$

11) $\lim\limits_{x\to0}\frac{arctg\,x}{x} = 1$

Exemple. Calculaţi limitele de funcţii:

1) $\lim\limits_{x\to\infty}\left(\frac{x+1}{x}\right)^x = 1^\infty = \lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

2) $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+1}\right)^x = 1^\infty = \lim\limits_{x\to\infty}\left(1+\frac{x-3}{x+1}-1\right)^x = \lim\limits_{x\to\infty}\left(1+\frac{-4}{x+1}\right)^x =$

$$= \lim_{x\to\infty}\left[\left(1+\frac{-4}{x+1}\right)^{\frac{x+1}{-4}}\right]^{\frac{-4}{x+1}\cdot x} = e^{\lim\limits_{x\to\infty}\frac{-4x}{x+1}} = e^{-4}$$

3) $\lim\limits_{x\to 0}(1+5x)^{\frac{1}{x}} = 1^{\infty} = \lim\limits_{x\to 0}\left[(1+5x)^{\frac{1}{5x}}\right]^{\frac{5x}{x}} = e^{\lim\limits_{x\to 0}\frac{5x}{x}} = e^{5}$

4) $\lim\limits_{x\to 0}\dfrac{\ln(1+10x)}{x} = \dfrac{0}{0} = \lim\limits_{x\to 0}\dfrac{\ln(1+10x)}{10x}\cdot 10 = 1\cdot 10 = 10$

5) $\lim\limits_{x\to 0}\dfrac{2^{x}-1}{x} = ln\,2$

6) $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x} = \dfrac{0}{0} = \lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x^2}\cdot\dfrac{x^2}{x} = \lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x^2}\cdot x = 1\cdot 0 = 0$

7) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-1}{x} = \dfrac{1}{3}$

8) $\lim\limits_{x\to 0}\dfrac{sin\,8x}{x} = \lim\limits_{x\to 0}\dfrac{sin\,8x}{8x}\cdot 8 = 1\cdot 8 = 8$

9) $\lim\limits_{x\to 0}\dfrac{tg\,x}{3x} = \lim\limits_{x\to 0}\dfrac{tg\,x}{x}\cdot\dfrac{1}{3} = 1\cdot\dfrac{1}{3} = \dfrac{1}{3}$

10) $\lim\limits_{x\to 0}\dfrac{arcsin\,x^2}{x^2} = 1$

11) $\lim\limits_{x\to 1}\dfrac{arctg(x-1)}{x^2-1} = \dfrac{0}{0} = \lim\limits_{x\to 1}\dfrac{arctg(x-1)}{(x-1)(x+1)} = 1\cdot\dfrac{1}{2} = \dfrac{1}{2}$