Profesor Blaga Mirela-Gabriela

SUME

1. $$\sum_{k=1}^{n} k\cdot k! = \sum_{k=1}^{n}(k+1-1)\cdot k! = \sum_{k=1}^{n}[(k+1)\cdot k! - k!] = \sum_{k=1}^{n}[(k+1)! - k!] = (n+1)! - 1$$

2. $$\sum_{k=1}^{n} k^2(k+1)! = \sum_{k=1}^{n}(k^2+4k+4-4k-4)(k+1)! =$$

$$\sum_{k=1}^{n}[(k+2)^2(k+1)! - (4k+4)(k+1)!] = \sum_{k=1}^{n}[(k+2)(k+2)! - (4k+4)(k+1)!] =$$

$$\sum_{k=1}^{n}[(k+2)(k+2)! - (k+1)(k+1)!] - 3\sum_{k=1}^{n}(k+1)(k+1)! \overset{1.}{\cong}$$

$$(n+2)(n+2)! - 4 - 3[(n+2)! - 2] = (n-1)(n+2)! + 2$$

3. $$\sum_{k=1}^{n}\frac{k}{(k+1)!} = \sum_{k=1}^{n}\frac{k+1-1}{(k+1)!} = \sum_{k=1}^{n}\left[\frac{k+1}{(k+1)!} - \frac{1}{(k+1)!}\right] = \sum_{k=1}^{n}\left[\frac{1}{k!} - \frac{1}{(k+1)!}\right] = 1 - \frac{1}{(n+1)!}$$

4. $$\sum_{k=1}^{n}\frac{n+k}{(n+k+1)!} = \sum_{k=1}^{n}\frac{n+k+1-1}{(n+k+1)!} = \sum_{k=1}^{n}\left[\frac{1}{(n+k)!} - \frac{1}{(n+k+1)!}\right] = \frac{1}{(n+1)!} - \frac{1}{(2n+1)!}$$

5. $$\sum_{k=1}^{n}\frac{1}{(k+1)!+k!} = \sum_{k=1}^{n}\frac{1}{(k+2)k!} = \sum_{k=1}^{n}\frac{k+1}{(k+2)!} \overset{3.}{\cong} \frac{1}{2} - \frac{1}{(n+2)!}$$

6. $$\sum_{k=1}^{n}\frac{k+2}{k!+(k+1)!+(k+2)!} = \sum_{k=1}^{n}\frac{k+2}{(k+2)^2k!} = \sum_{k=1}^{n}\frac{1}{(k+2)k!} \overset{5.}{\cong} \frac{1}{2} - \frac{1}{(n+2)!}$$

7. $$\sum_{k=0}^{n}\frac{k!}{(k+p)!} \underbrace{=}_{p\geq 2} \sum_{k=0}^{n}\frac{k!}{(k+p)(k+p-1)!} = \frac{1}{p-1}\sum_{k=0}^{n}\frac{k!\,(p-1)}{(k+p)(k+p-1)!} =$$

$$\frac{1}{p-1}\sum_{k=0}^{n}\frac{k!\,(k+p-1-k)}{(k+p)(k+p-1)!} = \frac{1}{p-1}\sum_{k=0}^{n}\left[\frac{k!\,(k+p)}{(k+p)(k+p-1)!} - \frac{k!\,(k+1)}{(k+p)(k+p-1)!}\right] =$$

$$\frac{1}{p-1}\sum_{k=0}^{n}\left[\frac{k!}{(k+p-1)!} - \frac{(k+1)!}{(k+p)!}\right] = \frac{1}{p-1}\left[\frac{1}{(p-1)!} - \frac{(n+1)!}{(n+p)!}\right]$$