FORMULE MATEMATICE

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = ax^2 + bx + c$,$a,b,c \in \mathbb{R}$, $a \neq 0$

$x_1,x_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ rădăcinile lui f

$V(-\frac{b}{2a}, -\frac{\Delta}{4a})$, $\min f/\max f = -\frac{\Delta}{4a}$, $\operatorname{Im} f = [-\frac{\Delta}{4a}, \infty)$,$a > 0$

forma canonică $f(x) = a\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a}$

$S = x_1 + x_2 = -\frac{b}{a}$ $\quad x_1^2 + x_2^2 = S^2 - 2P$

$P = x_1x_2 = \frac{c}{a}$ $\quad x_1^3 + x_2^3 = S^3 - 3PS$

$f(x) > 0 \iff a > 0, \Delta < 0$

semn

x	$-\infty$		x_1		x_2		∞
$ax^2 + bx + c$ $\Delta>0$		a	0	$-a$	0	a	
$\Delta=0$		a	0		a		
$\Delta<0$				a			

compunerea funcţiilor $\quad f \circ g(x) = f(g(x))$

f: $A \to B$ $\quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$f'(x) > 0$ sau < 0 $\quad \Rightarrow$ f injectivă (1)

$\forall\, y \in B\ \exists x \in A$ a.î. $f(x) = y$

$f(A) = B$ $\quad \Rightarrow$ f surjectivă(2)

(1),(2) $\Rightarrow$f bijectivă $\Rightarrow$

f inversabilă f: $A \to B$, $f(x) = y \Rightarrow f^{-1}:B \to A$, $f^{-1}(y) = x$

f: $A \to B$ $|A| = n$, $|B| = m$ $\quad$ numărul funcţiilor m^n

numărul funcţiilor injective A_m^n

f: $A \to A$ $|A| = n$ $\quad$ numărul funcţiilor bijective $n!$

$f(-x) = f(x)$ $\quad$ funcţie pară

$f(-x) = -f(x)$ $\quad$ funcţie impară

$f(x + T) = f(x)$, $T > 0$ $\quad$ funcţie periodică

$x \in \mathbb{R} \Rightarrow x = [x] + \{x\}$, $[x] \in \mathbb{Z}$, $\{x\} \in [0, 1)$ $\quad [x] \le x < [x] + 1$

$P_n = n!$, $A_n^k = \frac{n!}{(n-k)!}$, $C_n^k = \frac{n!}{k!(n-k)!}$, $0 \le k \le n$, $0! = 1$

$C_n^k = C_n^{n-k}$ (formula combinărilor complementare)

$(a+b)^n = C_n^0a^n + C_n^1 a^{n-1}b + \ldots + C_n^nb^n$(binomul lui Newton)

$T_{k+1} = C_n^k a^{n-k}b^k$ (formula termenului general)

$C_n^0 + C_n^1 + \ldots + C_n^n = 2^n$(nr. subm. unei mulţimi cu n elem.)

$C_n^0 + C_n^2 + \ldots = 2^{n-1} = C_n^1 + C_n^3 + \ldots$

$z = a + ib$ număr complex $a,b \in \mathbb{R}$, $\operatorname{Re} z = a$, $\operatorname{Im} z = b$, $i^2 = -1$

$\bar{z} = a - ib$

$|z| = \sqrt{a^2 + b^2}$, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$(\cos t + i\sin t)^n = \cos nt + i\sin nt$ (Moivre)

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\operatorname{tg} 2x = \frac{2\operatorname{tg}x}{1-\operatorname{tg}^2 x}$, $\operatorname{tg}(a+b) = \frac{\operatorname{tg}a+\operatorname{tg}b}{1-\operatorname{tg}a\operatorname{tg}b}$

$\sin(a \pm b) = \sin a\cos b \pm \sin b\cos a$

$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$

$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$

$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$

$\arcsin x + \arccos x = \frac{\pi}{2}$

$\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}$

$\sin x = a$, $a \in [-1,1]$ $\Rightarrow x = (-1)^k\arcsin a + k\pi$, $k \in \mathbb{Z}$

$\cos x = a$, $a \in [-1,1]$ $\Rightarrow x = \pm\arccos a + 2k\pi$, $k \in \mathbb{Z}$

$\operatorname{tg} x = a$, $a \in \mathbb{R}$ $\Rightarrow x = \operatorname{arctg} a + k\pi$, $k \in \mathbb{Z}$

$a^x = b \Leftrightarrow x = \log_a b$, $a \in (0, \infty)\setminus\{1\}$, $b>0$

$\log_a x = b \Leftrightarrow x = a^b$, $a \in (0, \infty)\setminus\{1\}$, $x>0$

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$\log_a 1 = 0$, $\log_a a = 1$, $\log_a a^n = n$, $\ln 1 = 0$, $\ln e = 1$, $\lg 10 = 1$

$\log_a x^n = n\log_a x$, $\log_{a^n} x = \frac{1}{n}\log_a x$, $\log_a x = \frac{1}{\log_x a}$

$\div$ $a_n = a_{n-1} + r$

$a_n = a_1 + (n-1)r$

$S_n = \frac{n(a_1+a_n)}{2} = \frac{n[2a_1+(n-1)r]}{2}$, $n = \frac{a_n-a_1+r}{r}$

$\div a,b,c \Longrightarrow 2b = a+c$

$\div\!\div$ $a_n = a_{n-1}\,q$, $a_1,q \neq 0$

$a_n = a_1q^{n-1}$

$S_n = a_1(1+q+q^2+\ldots+q^{n-1}) = a_1\frac{q^n-1}{q-1}$, $q \neq 1$ şi $S_n = na_1, q=1$

$\div\!\div$ $a,b,c \Longrightarrow b^2 = ac$

probabilitatea $P = \frac{\text{nr.cazurilor favorabile}}{\text{nr.cazurilor posibile}} \in [0, 1]$

Relaţiile lui Viéte

$ax^3+bx^2+cx+d=0$, $a \neq 0$, $x_{1,2,3}$ sunt rădăcinile ecuaţiei

$S_1 = x_1+x_2+x_3 = -\frac{b}{a}$

$S_2 = x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a}$

$S_3 = x_1x_2x_3 = -\frac{d}{a}$

$x_1^2 + x_2^2 + x_3^2 = S_1^2 - 2S_2$

x_1 rădăcină a ecuaţiei $\Rightarrow ax_1^3+bx_1^2+cx_1+d=0$

sau x_1 rădăcină a polinomului f $\Rightarrow f(x_1) = 0$

dacă $z_{1,2}$ sunt rădăcinile ecuaţiei $z^2+z+1=0$, atunci sunt şi rădăcinile ecuaţiei $z^3-1=0$ $\quad z^3-1 = (z-1)(z^2+z+1)$